LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2013**

MT 1817 - ORDINARY DIFFERENTIAL EQUATIONS

Date : 11/11/2013 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

Answer all questions. Each question carries 20 marks.

1. (a) Prove that $c_1t + c_2t^2 + c_3t^3$, $t \geq 0$ is a solution of $t^3x''' - 3t^2x'' + 6tx' - 6x = 0$. **(5)**

(OR)

(b) If Wronskian of two functions $x_1(t)$ and $x_2(t)$ on *I* is non-zero for atleast one point on the interval I, prove that $x_1(t)$ and $x_2(t)$ are linearly independent on *I*. **(5)**

(c) Find the general solution of $x'''(t) - x'(t) = e^t$, using method of variation of parameters. **(15)**

(OR)

(d) Discuss the various solutions of the second order linear homogenous equation with constant coefficients. **(15)**

2. (a) State and prove Rodrigue's formula. **(5)**

(OR)

(b) With usual notation, prove the following:

(i) $P_l'(1) = \frac{l(l+1)}{2}$

(ii) $P_{2l}(0) = (-1)^l \frac{(2l)!}{2^{2l}(l!)^2}$ **(5)**

(c) Solve by Frobenius method, $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$. **(15)**

(OR)

(d) Show that the generating function for the Legendre polynomial is $\frac{1}{\sqrt{1-2tx+t^2}} = \sum_{n=0}^{\infty} t^n P_n(x)$ if $|t| < 1$ and $|x| \leq 1$. **(15)**

3. (a) Prove that $J_{(-n)}(x) = (-1)^n J_n(x)$, where *n* is an integer. **(5)**

(OR)

(b) Prove that $e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. **(5)**

(c) Solve the Bessel's equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$. **(15)**

(OR)

(d) Prove that $J_n(x)$ and $Y_n(x) = \frac{cosn\pi J_n(x) - J_{(-n)}(x)}{sinn\pi}$ are two independent solutions of Bessel's equation for all values of n **(15)**

4. (a) Prove that all the eigen values of Strum-Liouville problem are real. **(5)**

(OR)

(b) State and prove Gronwall inequality. **(5)**

(c) State and prove Picard's theorem for initial value problem. **(15)**

(OR)

(d) Let $G(t, s)$ be the Green's function. Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0$, $a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$. **(15)**

5. (a) Illustrate asymptotically stable solution by an example. **(5)**

(OR)

(b) Prove that the null solution of equation $x' = A(t)x$ is stable if and only if a positive constant k exists such that $|\phi(t)| \leq k, t \geq t_0$. **(5)**

(c) Discuss the stability of non-autonomous systems. **(15)**

(OR)

(d) Explain the stability of $x' = Ax$ by Lyapunov's method. **(15)**